THEORETICAL STUDY OF FLOW ABOUT SINGLE AND DOUBLE RECESSES IN A PLANE CHANNEL

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A two-parameter model of turbulence is used to calculate flow characteristics in a channel with a sudden expansion in the form of two recesses with in the expansion range 1.3-4.0. The limits of applicability of the Borda formula are evaluated.

Turbulent separated flows after recesses in pipes and channels are of great interest to investigators. Such flows have been studied the most in circular pipes. The work [1] presented detailed measurements of velocities and turbulence characteristics in a circular pipe with a sudden expansion. Data on the static pressure distribution on a pipe wall was communicated in [2, 3]. A survey of studies of plane flows beyond recesses was offered in [4]. Several works have presented information on the length of the separation zone and the distribution of the mean velocities and turbulence characteristics. Pressure distributions on the walls have been reported mainly for the case of a recess in a free flow. There has been less study of flow in a plane channel with a sudden expansion in the form of two recesses symmetrical relative to the middle plane. We know only of the work of Abbott and Klein [5], which represents an expanded experimental study of flows of an incompressible liquid in a channel with a single-stage and two-stage expansion of degrees from 1.1 to 5. Measurements of the lengths of the separation zones were obtained by the method of visualization with the aid of an oil film. The authors also present a small number of mean-velocity profiles and, for the case of a single recess, turbulence intensities.

Presented below are results of a theoretical study of the turbulent flow of an incompressible liquid in a plane channel with a sudden expansion in the form of one and two recesses.

1. The system of equations for the steady turbulent motion of an incompressible liquid has the form [6]

$$\frac{U\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{\partial P}{\partial x}+2\frac{\partial}{\partial x}\left[(\nu_t+\nu)\frac{\partial U}{\partial y}\right]+\frac{\partial}{\partial y}\left[(\nu_t+\nu)\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\right],$$
$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial y}+\frac{\partial}{\partial x}\left[(\nu_t+\nu)\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\right]+2\frac{\partial}{\partial y}\left[(\nu_t+\nu)\frac{\partial V}{\partial y}\right], \quad (1)$$
$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0.$$

To determine ν_t, it was proposed in [7] that a two-parameter turbulence model k—ε be used (k is the eddy kinetic energy and ε is the rate of dissipation of the eddy kinetic energy):

$$\frac{U\partial k}{\partial x}+V\frac{\partial k}{\partial y}=\frac{\partial}{\partial x}\left[\left(\frac{\nu_t}{\sigma_k}+\nu\right)\frac{\partial k}{\partial x}\right]+\frac{\partial}{\partial y}\left[\left(\frac{\nu_t}{\sigma_k}+\nu\right)\frac{\partial k}{\partial y}\right]+S_k,$$
$$\frac{U\partial \varepsilon}{\partial x}+V\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial x}\left[\left(\frac{\nu_t}{\sigma_\varepsilon}+\nu\right)\frac{\partial \varepsilon}{\partial y}\right]+\frac{\partial}{\partial y}\left[\left(\frac{\nu_t}{\sigma_\varepsilon}+\nu\right)\frac{\partial \varepsilon}{\partial y}\right]+S_\varepsilon, \quad (2)$$
$$\nu_t=C_D\frac{k^2}{\varepsilon}.$$

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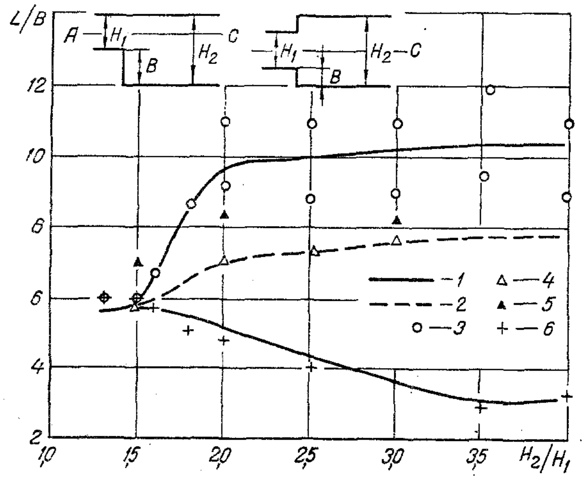

Fig. 1. Dependence of the lengths of the separation zones on the degree of expansion: 1 and 2) double and single recesses, respectively; 3, 4, and 6) experiment [5] (3, 6 are for a double recess, 4 is for a single recess); 5) single recess, measurements obtained with a laser Doppler anemometer [4].

Here, S represents source terms of the transfer equations:

$$S_k = \nu_t F_k - \varepsilon;$$

$$S_\varepsilon = C_{\varepsilon 1}\frac{\varepsilon}{k}\nu_t F_k - C_{\varepsilon 2}\frac{\varepsilon^2}{k};$$

$$F_k = 2\left(\frac{\partial U}{\partial x}\right)^2 + 2\left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)^2;$$

$\sigma_k = 1$; $\sigma_\varepsilon = 1.3$; $C_D = 0.09$; $C_{\varepsilon_1} = 1.44$; $C_{\varepsilon_2} = 1.92$ are constants of the model.

In the process of solving Eqs. (1), we eliminated the pressure from them by introducing vorticity Ω, a variable of the stream function ψ, so that

$$U = \frac{\partial\psi}{\partial y},\ V = -\frac{\partial\psi}{\partial x},\ \Omega = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}, \tag{3}$$

and then chose a finite-difference scheme [8]. The resulting system of algebraic equations was solved by the Gauss–Seidel method.

We assigned values of the stream function as the boundary condition on the impermeable walls. For the other variables, the boundary conditions were taken one cell of the difference grid deeper into the flow, where we assumed satisfaction of the universal "wall law" [6]:

$$\frac{U}{u^*} = \frac{1}{\varkappa}\ln\frac{u^* n}{\nu} + A,\ k = \frac{u^{*2}}{\sqrt{C_D}},\ \varepsilon = \frac{u^{*3}}{\varkappa n}, \tag{4}$$

here $\varkappa = 0.41$; $A = 5.36$; u^* is the dynamic velocity; n is the distance to the wall.

The values of ψ, Ω, k, and ε were assigned in the inlet section of the channel, while "mild" boundary conditions were set for the outlet section: the longitudinal derivatives of all of the dependent variables were qual to zero.

Near the nodal point beyond the sudden expansion the flow is turbulent, so that the logarithmic law (4) was not used at points near this nodal point. In its place we set the condition of a drop in the streamline from the nodal point in the horizontal direction. We took the same value for the turbulence energy as in the boundary layer of the incoming flow. We found ε by using the condition of proportionality of the turbulence scale in the mixing layer to the width of the layer [9]. After finding ψ, Ω, k, and ε, we used Eqs. (1)

to find the pressure gradient; the pressure was determined by calculating the integral beginning from the inlet section along the line AC (Fig. 1) of the channel and then perpendicular to it up to the walls. We used a uniform finite-difference grid. Systematic calculations performed with refinement of the grid showed that cells corresponding in size to $(1/10)H_2$ along the channel and $(1/20-1/30)H_2$ over the channel height are sufficient. The calculations were performed on a BÉSM-6 computer.

2. Figure 1 shows the results of our theoretical study in comparison with experimental data from [5]. The results are presented in the form of the dependence of the relative lengths of the separation zones L/B (B is the depth of the recess) on the value of H_2/H_1, characterizing the degree of expansion of the channel. It can be seen that the length of the separation zone is nearly constant in the case of a single recess and amounts to 7-8 recess depths. However, in the case of a channel with an expansion in the form of two recesses and a degree of expansion greater than 1.5, the average flow becomes nonsymmetrical. As the degree of expansion is increased, the flow approaches one wall and creates a separation zone with markedly different lengths (on the order of 10B and (3-4)B, at the walls. In the experiments in [5], the thickness of the boundary layer in the inlet channel was 1/4 of the channel height, while the Reynolds number of the incoming flow $U_1H_1/\nu = 2 \cdot 10^4-5 \cdot 10^4$. The calculations were performed with different thicknesses of the boundary layer at the inlet: from a uniform flow to a fully developed turbulent flow, and within the range of Reynolds numbers 10^4-10^6. Branching of the symmetrical solution occurred regardless of the thickness of the boundary layer or the Reynolds number when $H_2/H_1 = 1.5$. The spread of empirical data in Fig. 1 for the "long" separation zone is connected with low-frequency fluctuations of the length of the zone in the case of a high degree of channel expansion. No transverse fluctuations were seen in the experiments. On the average, the theoretical separation-zone lengths were close to the experimental values, which justifies the use of the steady-state equations of motion in the calculation.

3. Among the flow characteristics which are important for engineering applications are the coefficient of static-pressure recovery $\overline{C}_P$ and the coefficient of total-pressure loss ζ in the channel:

$$\overline{c}_P = \frac{\overline{P}_2 - \overline{P}_1}{1/2U_1^2}, \quad \zeta = \frac{\overline{P}_1 + 1/2\alpha_1 U_1^2 - \overline{P}_2 - 1/2\alpha_2 U_2^2}{1/2U_1^2} = \alpha_1 - \overline{c}_P - \alpha_2 \frac{U_2^2}{U_1^2}. \tag{5}$$

In these formulas we average over the mass rate:

$$\overline{P} = \frac{\int_{(S)} P dm}{m}, \tag{6}$$

S is the cross-sectional area of the channel, m is the mass rate, and α is the energy coefficient of the velocity profile:

$$\alpha = \frac{\int_{(S)} U^2 dm}{m} \frac{S^2}{m^2}. \tag{7}$$

In determining $\overline{C}_P$ and ζ, it is usually assumed that the flows in the inlet and outlet sections of the channel are uniform and parallel and that the static pressure in the section of the sudden expansion is constant. Then the below Borda—Carnot formula follows from the theorem of the conservation of momentum and mass, without allowance for friction on the walls [10]:

$$\overline{c}_P = 2\frac{H_1}{H_2}\left(1 - \frac{H_1}{H_2}\right), \quad \zeta = \left(1 - \frac{H_1}{H_2}\right)^2. \tag{8}$$

Figure 2 shows calculated distributions of the coefficients of static-pressure recovery and total-pressure loss along the channel with a uniform velocity profile at the inlet and a Reynolds number of 10^5. The distance from the section of sudden expansion is referred to the depth of the recess. For control purposes, Fig. 2a shows results of measurements of the static pressure on the wall of a channel after a single recess [11]. Since the pressure changes little over the channel height after the flow reattachment point, the coefficient

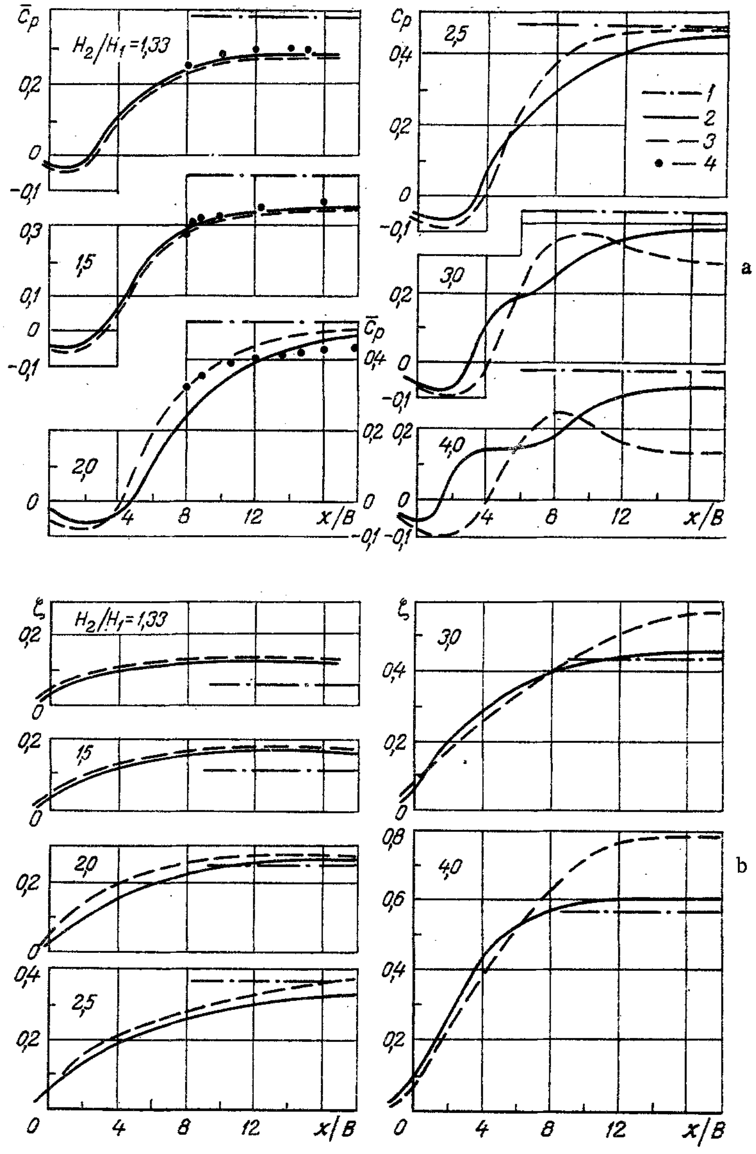

Fig. 2. Theoretical change in the coefficient of static-pressure recovery (a) and the coefficient of total-pressure loss (b) along the channel: 1) Borda formula; 2) double recess; 3) single recess; 4) pressure at the wall of a channel beyond a single recess, experiment in [11].

of pressure recovery on the wall is close to the average. It is apparent that the calculation quite accurately predicts the pressure recovery beyond the zone of reversed currents. It should be noted that static pressure begins to decrease before the separation. The minimum value of $\overline{C}_p$ is reached at $x/B = 1.5–2$ and $\overline{C}_p$ passes through zero at $x/B = 3–4$.

Due to the high levels of turbulence, the velocity profiles beyond the flow separation zone are almost equilibrium profiles (the energy coefficients of the profiles, shown in Fig. 3a, are close to unity), so that $\overline{C}_p$ rapidly increases to the maximum value. For the case of a single recess with $H_2/H_1 > 1.5$, pressure in the section of the sudden expansion is almost constant (Fig. 3c), so that the losses due to the shock are close to those calculated by Eq. (8). When $H_2/H_1 \leqslant 1.5$, the maximum value of $\overline{C}_p$ decreases due to a drop in bottom pressure as a result of curvature of the streamlines in the region above the separation zone and it approaches the values measured beyond the recess in the free flow. When $H_2/H_1 \geqslant 3$, the maximum of $\overline{C}_p$ decreases due to an increase in the loss connected with the high relative levels of turbulence. When $H_2/H_1 \leqslant 2.5$, the zone of nearly constant $\overline{C}_p$ extends far down-

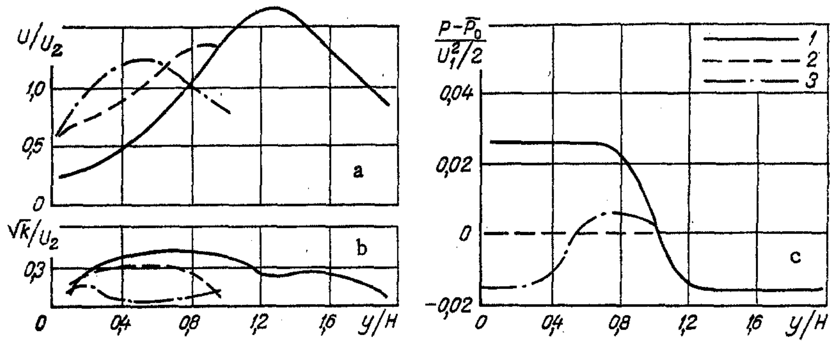

Fig. 3. Flow characteristics with $H_2/H_1 = 1.33$ and $H_2/H_1 = 3$: distributions of the longitudinal velocity component at the section $x/B = 12$ (a), turbulence energy at the section $x/B = 12$ (b), static pressure at the section of the sudden expansion (c); 1) double recess, $H_2/H_1 = 3$; 2 and 3) single recesses, $H_2/H_1 = 3$ and $H_2/H_1 = 1.33$, respectively.

stream, while when $H_2/H_1 > 2.5\bar{C}_p$ reaches its maximum near the reattachment point and then decreases due to the large losses to turbulent friction. Thus, the Borda–Carnot formula is quite accurate within the narrow range $2 \leqslant H_2/H_1 \leqslant 2.5$. Now let us consider a channel with a double recess. In this case, the static pressure in the section of sudden expansion is not constant for high degrees of expansion but instead increases at the end corresponding to the "long" separation zone (Fig. 3c). Thus, the losses due to the shock decrease compared to the case of a single recess, although the friction losses are high. A decrease in pressure at the wall toward which the flow is compressed increases the stability of the asymmetric flow. The flow expands in two stages in such a channel, which is quite evident from Fig. 2a: First there is a "short" separation zone at the wall and then a "long" separation zone, as though modeling the replacement of a double recess by two successive single recesses. This reduces the losses due to shock. Thus, the coefficient of static-pressure recovery and total-pressure loss is higher and the rate of increase in the loss lower in a channel with a double recess than in a channel with a single recess.

Consequently, given the same degree of expansion, a sudden expansion in the form of two recesses located symmetrically relative to the middle plane is more efficient than a sudden expansion in the form of a single recess.

NOTATION

x, y, Cartesian coordinates; U, V, mean velocity components; P, static pressure; H_1, H_2, height of channels before and after the sudden expansion; U_1, U_2, mean velocities in channels of height H_1 and H_2; $H = H_2$ for a single recess and $H = (1/2)H_2$ for a double recess; U_m, maximum velocity in the channel of height H_1; B, depth of recess; $\bar{P}_0$, mass-rate-mean pressure in the section of the sudden expansion; $\bar{C}_p$, coefficient of static-pressure recovery; ζ, coefficient of total-pressure loss; k, eddy kinetic energy; ε, rate of dissipation of eddy kinetic energy; ν, viscosity.

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STABILITY LIMIT OF THERMALLY DRIVEN OSCILLATIONS IN A TUBE OF VARIABLE CROSS SECTION

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The stability limit for helium in a tube of variable cross section is established and experimentally confirmed.

As is known, in a nonisothermal tube with a closed heated end and the other end placed in a cryostat with liquid helium, thermally driven oscillations may occur. Here, the heat flow to the liquid helium may increase by an order or more [1, 2].

One method of studying this phenomenon is determining the range of parameters of the system within which these oscillations may take place, i.e. finding the necessary condition for occurrence of the oscillations or solutions of the stability problem. This problem was examined in [3-6] for tubes of constant cross section. The present work studies the stability of oscillations in tubes the radius of which changes along their length and, in particular, increases intermittently. This corresponds to a tube composed of tubes of different diameter or to the attachment of closed volumes to the ends of a tube. The model and methods developed in [3, 4] will be used to construct the mathematical model and solve the problem.

These works made assumptions regarding triviality and did not consider: 1) the radial gradient of the acoustic pressure; 2) the radial change in the mean temperature and viscosity; 3) heat flow and friction due to axial gradients. For a tube of variable cross section, we add the assumptions that the section of an elemental streamtube changes in proportion to the section of the tube, i.e. $r(x)\Delta r(x) \sim r_0^2(x)$, and that the radial component of the velocity of the gas can be ignored. The latter assumption is valid for tubes with a small change in section along their length or when the absolute value of the velocity is low and, thus, its radial component is small (for example, in the case of a sudden contraction or expansion of the tube section near the closed end). Allowing for these assumptions, the linearized system of equations for the tube of variable cross section has the form

$$\frac{\partial \rho}{\partial \tau} + \frac{\rho_0}{r_0^2(x)} \frac{\partial}{\partial x} (r_0^2(x) U) + \frac{\partial \rho_0}{\partial x} U = 0, \tag{1}$$

$$\frac{\partial U}{\partial \tau} + \frac{1}{\rho_0} \frac{\partial P}{\partial x} = \nu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right), \tag{2}$$

$$\frac{\partial T}{\partial \tau} + U \frac{dT_0}{dx} - \frac{1}{\rho_0 c_p} \frac{\partial P}{\partial \tau} = \frac{\lambda}{\rho_0 c_p} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right). \tag{3}$$

The system is closed by the linearized equation of state

$$T = \frac{P}{R\rho_0} - \frac{T_0}{\rho_0} \rho. \tag{4}$$

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